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# UW30. Gradient based processing for linear vector sensor

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The demand for higher performance arrays, coupled with recent advancements in vector sensor technology, has resulted in a surge in research on vector sensor based arrays in the past decade. In this paper, a new approach to vector sensor array processing is proposed. The method relies on extracting the so called 'acoustic modes' of the propagating wavefield using the spatial gradients of the field variables. These gradients are approximated with a finite difference scheme along a uniform linear array of vector sensors. The proposed method is evaluated through a series of numerical simulations. The resulting broadband beamformer is shown to have a directivity factor comparable to conventional beamformers with much larger apertures while providing all the advantages of acoustic vector sensors such as port-starboard discrimination. Owing to the sufficiency of shorter apertures and other potential operational advantages, the proposed method is suitable for use in short towed arrays with autonomous underwater platforms.

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# 1 INTRODUCTION

The conventional approach for passive underwater acoustic surveillance is to use uniform linear arrays (ULA) that consist of several nested sub-arrays of pressure measuring omni-directional hydrophones towed behind an underwater or surface platform. Towed arrays have a wide range of applications including underwater warfare, geophysical exploration, marine life research, and underwater acoustic communications. In such arrays, hydrophones are placed at a distance equal to half the wavelength (of the design frequency) apart to achieve an optimal array response in terms of main lobe width and suppressing grating sidelobes. Thus, a typical ULA can have an aperture length of 200 m or more (Barbagelata et al., 2008). In addition, use of omni-directional hydrophones results in port-starboard ambiguity.

The demand for higher performance towed array systems, coupled with recent advancements in sensor technology, has resulted in a surge in research on non-conventional towed arrays. In one alternative type of ULA, each omni-directional hydrophone in the array is replaced with a set of three omni-directional hydrophones (i.e., triplets) to form a cardioid response pattern at each triplet. The cardioid's null can electronically be placed to the port or starboard, overcoming the port-starboard ambiguity problem (Hughes, 2000). ULAs that incorporate vector sensors capable of measuring the acoustic particle velocity and pressure (rather than pressure measuring hydrophones) are also capable of forming first order (Craig and Nuttall, 2001) and higher order (Smith and van Leijen, 2007) cardioid shaped response patterns at each sensor; thus, eliminating the port-starboard ambiguity and improving array directivity compared to conventional ULAs.

With the advancements in underwater robotics technology, towed arrays are more commonly being deployed off autonomous underwater vehicles (AUV) which have much lower operating costs compared to manned platforms. However, the limited available onboard power and thrust of AUVs necessitates much smaller sized towed arrays. Several such towed arrays specifically designed for AUV platforms have been developed over the last decade. These hydrophone based thin line arrays typically have a length between 10-15 m and an outer diameter of 1-2 cm (Holmes et al., 2006; Pallayil et al., 2007; Maguer et al., 2009). With this paper, a novel array processer for vector sensor based thin line arrays is proposed.

# 2 THEORETICAL DEVELOPMENT

## 2.1 The Acoustic Pressure Field

Consider a ULA of particle velocity sensors along the *x*-axis of a Cartesian coordinate system. In addition to the particle velocity sensors, an omni-directional pressure sensor is placed at the center of the array as shown in Figure 1. Assume that there is an incident time-harmonic plane wave with a propagation vector $-\mathbf{k}_i$ and angular frequency of $\omega = k_i \cdot c$ where $k_i = |\mathbf{k}_i|$.

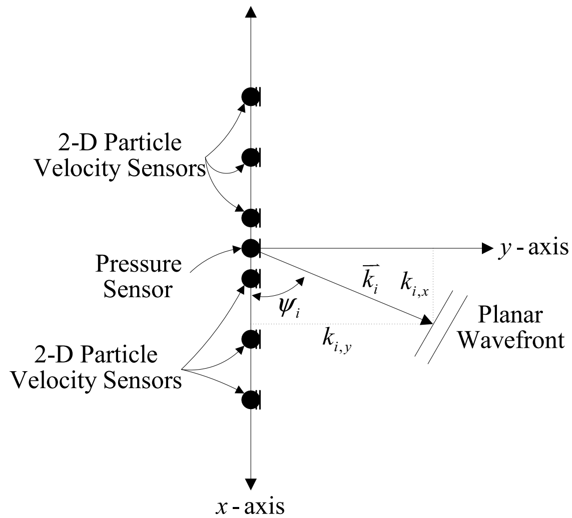


Figure 1. The configuration of the vector sensor array with respect to the Cartesian coordinate system.

The pressure and particle velocity at any point $x$ along the array (omitting the time dependent part, i.e., $\exp(j\omega t)$) can be expressed as:

$$\begin{aligned} p(x) &= P\exp(jk_i\cos\psi_i x), \\ v_x(x) &= V\cos\psi_i\exp(jk_i\cos\psi_i x), \\ v_y(x) &= V\sin\psi_i\exp(jk_i\cos\psi_i x). \end{aligned} \tag{1}$$

where $P$ and $V$ are the amplitudes of the pressure and velocity field variables, and $\psi_i$ is the azimuth angle of incidence of the incoming plane wave. For the particle velocities in Eq. (1), the spatial gradients can be expressed as:

$$\begin{aligned} \frac{\partial^n v_x}{\partial x^n} &= V\left(jk_i\right)^n\cos\psi_i\left(\cos\psi_i\right)^n\exp(jk_i\cos\psi_i x), \\ \frac{\partial^n v_y}{\partial x^n} &= V\left(jk_i\right)^n\sin\psi_i\left(\cos\psi_i\right)^n\exp(jk_i\cos\psi_i x). \end{aligned} \tag{2}$$

Evaluating the $n^{\text{th}}$ order gradients at the center of the array (i.e., at $x=0$) results in:

$$\begin{aligned} \frac{\partial^n}{\partial x^n}v_x(0) &= V\left(jk_i\right)^n\left(\cos\psi_i\right)^{n+1}, \\ \frac{\partial^n}{\partial x^n}v_y(0) &= V\left(jk_i\right)^n\sin\psi_i\left(\cos\psi_i\right)^n. \end{aligned} \tag{3}$$

## 2.2 Approximating Gradients with Finite Differences

To approximate the gradients given in Eq. (3) with the sensor measurements, define the finite difference operator:

$$g_n[\nu(\xi)] = \frac{\delta_n[\nu(\xi)]}{d^n}, \tag{4}$$

where $\nu(\xi)$ is any acoustic field variable at coordinate $\xi$ along the x-axis, $n$ is the order of the gradient, $d$ is the separation between the sensors, and the $\delta_n(\cdot)$ operator is defined as:

$$\delta_n[\nu(\xi)] = \sum_{l=0}^{n}(-1)^{n-l}\binom{n}{l}\nu\left(\xi + \left(l - n/2\right)d\right). \tag{5}$$

The coefficients of the $\delta_n(\cdot)$ operator are the binomial coefficients:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \tag{6}$$

For small sensor spacing relative to the wavelength (i.e., $k_i d \ll 1$), the $n^{\text{th}}$ order finite difference operation $g_n(\cdot)$ on an acoustic field variable approximates the $n^{\text{th}}$ order gradient of that field variable evaluated at the origin (e.g., $\partial^n v_x(0)/\partial x^n \approx g_n[v_x(0)]$). Table I outlines the parameters of the finite difference operator $g_n(\cdot)$ used for approximating the first five gradients of the acoustic field variables.

Table I. The parameters of the finite difference operator $g_n(\cdot)$ used for approximating the first few gradients of the acoustic field variables.

| Gradient Order $(n)$ | Required Measurement Locations ($\xi = 0$) $(k - n/2)$ | Corresponding Binomial Coefficients $(-1)^{(n-k)} \cdot [n!/(k!\cdot(n-k)!)]$ |
|---|---|---|
| 0 | 0 | 1 |
| 1 | $+d/2, -d/2$ | $+1, -1$ |
| 2 | $-d,\ 0, +d$ | $+1, -2, +1$ |
| 3 | $+3d/2,\ +d/2, -d/2,\ -3d/2$ | $+1, -3, +3, -1$ |
| 4 | $+2d,\ +d,\ 0, -d, -2d$ | $+1, -4, +6, -4, +1$ |
| 5 | $+5d/2,\ +3d/2,\ +d/2,$ $-d/2,\ -3d/2, -5d/2$ | $-1, +5, -10, +10, -5, +1$ |

## 2.3 Extracting Acoustic Modes from the Gradient Estimates

The acoustic field variable gradients defined in Eq. (2) can be approximately obtained from the measurements at various sensors using the finite difference operator $g_n(\cdot)$. Note that these gradients for the *x*- and *y*-direction particle velocities involve the trigonometric terms $(\cos\psi_i)^{n+1}$ and $\sin\psi_i(\cos\psi_i)^n$, respectively. The $n^{\text{th}}$ spatial gradient of the *x*-direction particle velocity evaluated at the origin $x=0$ (given in Eq. (3)) can be expanded as:

$$\frac{\partial^n}{\partial x^n}v_x(0)=V\left(jk_i\right)^n\cdot\begin{cases}\left[\dfrac{2}{2^{n+1}}\displaystyle\sum_{k=0}^{n/2}\binom{n+1}{k}\cos\left[(n-2k+1)\psi_i\right]\right], & n \text{ even}\\ \left[\dfrac{2}{2^{n+1}}\displaystyle\sum_{k=0}^{(n-1)/2}\binom{n+1}{k}\cos\left[(n-2k+1)\psi_i\right]+\dfrac{1}{2^{n+1}}\binom{n+1}{(n+1)/2}\right], & n \text{ odd,}\end{cases} \tag{7}$$

where the $\cos[(n-2k+1)\psi_i]$ terms are the cosine modes. Likewise, the $n^{\text{th}}$ spatial gradient of the *y*-direction particle velocity evaluated at the origin can be expanded as:

$$\frac{\partial^n}{\partial x^n}v_y(0)=V\left(jk_i\right)^n\cdot\begin{cases}\left[\dfrac{1}{2^n}\displaystyle\sum_{k=0}^{n/2}\left[\dfrac{n!(n-2k+1)}{k!(n-k+1)!}\right]\sin\left[(n-2k+1)\psi_i\right]\right], & n \text{ even}\\ \left[\dfrac{1}{2^n}\displaystyle\sum_{k=0}^{(n-1)/2}\left[\dfrac{n!(n-2k+1)}{k!(n-k+1)!}\right]\sin\left[(n-2k+1)\psi_i\right]\right], & n \text{ odd,}\end{cases} \tag{8}$$

where the $\sin[(n-2k+1)\psi_i]$ terms are the sine acoustic modes.

## 2.4 The Velocity Gradient Beamforming Algorithm

As indicated in Table I, measurement points necessary to approximate the even and odd ordered gradients do not coincide. This suggests that to be able to approximate up to the $N^{\text{th}}$ order gradient, one needs $2M-1$ sensors separated by a distance of $d/2$. However, as pointed out by Franklin (1997), shifting the reference center for the even numbered modes by a distance of $d/2$ along the negative *x*-direction, it is possible to use the same sensors for estimating both the even and odd numbered gradients. Assuming that the incident wave is a plane wave, the shift in the center of reference for the even ordered gradients can be corrected in the post-processing stage by introducing a time delay. As a consequence, it is possible to estimate up to the $N^{\text{th}}$ order gradient using $M=N+1$ sensors separated by a distance of $d$.

Once the gradients are computed from finite differencing as outlined in Section 2.2, each gradient is weighted by a normalization factor $a_n=(jk_i)^{-n}$ (to compensate for the term appearing in Eq.(3)) and a filter coefficient $w_{x,n}$ (for *x*-velocity gradients) or $w_{y,n}$ (for *y*-velocity gradients), and summed. Since the contribution of each gradient order to the acoustic modes is readily available from Eq. (7) and Eq. (8), it is possible to obtain a least squares solution for the filter weights $w_{x,n}$ and $w_{y,n}$ that result in a beamformer response of the form:

$$r(\psi) = \sum_{n=0}^{N+1} \cos(n\psi). \tag{9}$$

# 3 NUMERICAL RESULTS

## 3.1 Beampatterns

Theoretical beampatterns at different steer angles for an array of six vector sensors capable of measuring the two orthogonal components of the particle velocity field are shown in Figure 2. It can be observed from these plots that the beampattern is not distorted as steered from the broadside (Figure 2 (d)) towards the endfire condition (Figure 2 (b)).

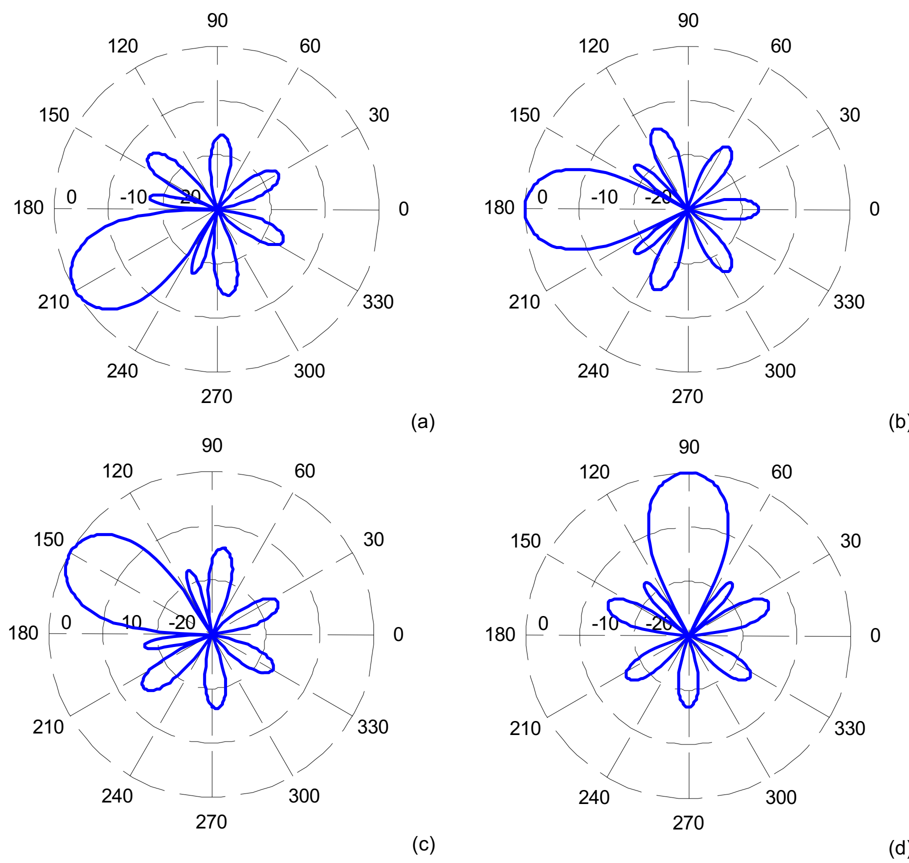


Figure 2. The beampattern of a short array of six 2-D particle velocity sensors steered towards (a) 210, (b) 180 (endfire), (c) 150, and (d) 90 (broadside) degrees, respectively.

## 3.2 Directivity

The directivity of the proposed beamformer can be formulated as:

$$D(\psi) = \frac{1}{2(M+1)} + \frac{\sin[(M+1/2)\psi]}{2(M+1)\sin(\psi/2)}. \quad (10)$$

The half power beamwidth was numerically calculated as a function of the number of sensors and is plotted in Figure 3. The theoretical result in Eq. (10) suggests that the beampattern and array directivity are independent of frequency.

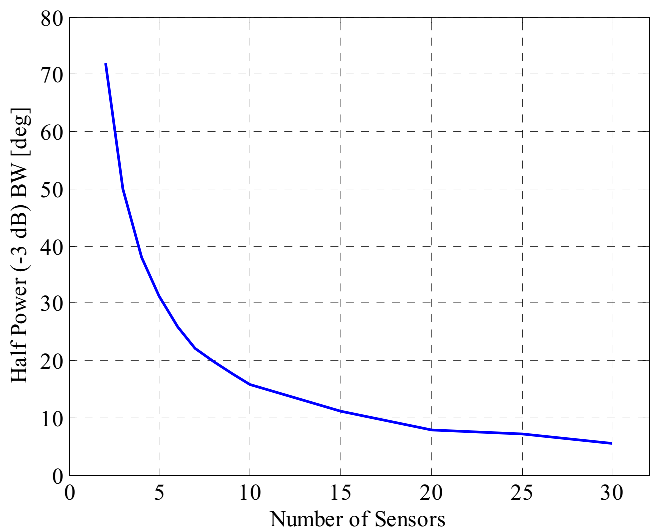


Figure 3. The half power beamwidth as a function of the number of sensors *M* in the array.

## 3.3 Theoretical Performance in Various Noise Fields

The array response was also evaluated under several theoretical noise fields including 2-D isotropic noise and dipole noise (representative of sea surface noise). For this purpose, Monte-Carlo simulations were conducted using artificially generated time-harmonic plane waves and noise signals with various signal to noise ratios. The array response obtained from one such simulation using dipole noise for different number of averages (each average was obtained from 160 ms of data) is shown in Figure 4. These simulations revealed that the beamformer is sensitive to the type of noise and SNR. Further research is being conducted to quantify the response of the processor under different types of noise.

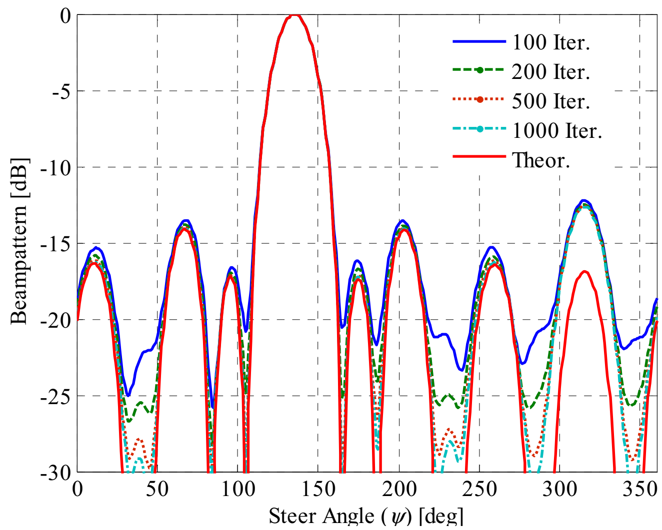


Figure 4. A Monte-Carlo simulation result obtained for dipole noise for an array of $M = 6$ sensors.

# 4 CONCLUSIONS

A novel vector sensor array processing scheme is introduced in this paper. The proposed planar beamformer estimates the gradients of the particle velocities from the measurements based on a finite differencing scheme. These gradients are then used to extract the directional modes of the acoustic field and to form the beamformer output. The accuracy of the gradient estimates is dependent on the sensor separation which must satisfy (in contrast to for conventional ULAs). The main beamwidth of the proposed method is observed to be very similar to that of the conventional delay-and-sum beamformer using equal number of sensors. Thus, the proposed approach enables very short aperture lengths compared to conventional beamformers, making it suitable for AUV based towed arrays. Other advantages of the gradient based method are linearity, improved port-starboard discrimination ability, and frequency independent response. In addition, the beampattern is not significantly distorted as it is steered from broadside to endfire. At the time of the writing of this paper, the sensitivity of the beamformer to various noise fields was being investigated. In parallel, in-air experiments are being conducted using an array of six 2-D particle velocity sensors. The extension of the planar processor to 3-D is not straightforward and is currently being investigated.

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