Wavelet Domain Estimation of Frequency Response Functions

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INTRODUCTION

FFT based estimates of frequency response functions (FRF) are the most widespread means of experimentally evaluating the characteristics of vibrating structures. However, it is well known that such FRF estimates are highly sensitive to measurement noise and usually require several ensemble averages for stable, noise-free estimates. In experimental analysis, the ensemble averages are not available and are replaced with time averages which require that both the input and system be stationary within the observation interval. This assumption may not necessarily be true for real-life non-stationary systems and as more averages are taken, the less accurate the FRF estimates become.

Although the wavelet transform was initially developed for time-frequency analysis, as an alternative to the short time frequency transform (STFT), it has found several other important application areas. One such application is the wavelet domain noise reduction which has been successfully applied in many disciplines such as audio and medical image restoration. This paper addresses wavelet domain noise reduction of FRFs as an alternative to time averaging FFT methods with the expectation that comparable estimation accuracy can be achieved with a fewer number of samples.

THEORY

The input-output relationship of a linear, time-invariant (LTI) system is given through the convolution integral:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)u(\tau)d\tau \tag{1}$$

where $y(t)$, $u(t)$, and $h(t)$ are the output, input, and system impulse response, respectively. The true FRF of the system, $H(\omega)$ is defined as the Fourier transform of the system impulse response. The FRF can be estimated through the ratio of the cross- and auto-power spectral density (PSD) of the input and output. The PSD is generally estimated using periodograms and it is well known that the periodogram is an asymptotically unbiased but inconsistent estimator of the PSD. Therefore, increasing the number of samples used in calculating the periodograms reduces the PSD bias, but may not sufficiently improve the variance of the FRF estimates. Rather, time averages of FRF estimates are computed in order to reduce the associated variance.

FRF estimates are also affected by measurement noise at the input and outputs. In the presence of measurement noise, the convolution integral becomes:

$$z(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau + v(t) \tag{2}$$

where $x(t) = u(t) + w(t)$ is the noisy input signal. While modeling measurement noise as a Gaussian process [i.e., $v(t) \sim \mathcal{N}(0,\sigma_v^2)$; $w(t) \sim \mathcal{N}(0,\sigma_w^2)$] is a reasonable assumption, the resulting noise in the FRFs is highly impulsive and cannot be modeled as additive Gaussian noise. Based on empirical observations, FRF noise is modeled as a complex symmetric alpha stable distribution, particularly as a complex Cauchy distribution.

The discrete wavelet transform (DWT) is defined as the integral transform:

$$\{\mathcal{W} f\}(j,k) = \int_{-\infty}^{\infty} f(t)2^{-j/2}\psi(2^{-j}t-k)dt \tag{3}$$

where $f(t)$, $\psi(t)$, j and k are the transformed function, wavelet basis function, scale and translation variables, respectively. The scale variable is inversely proportional to the frequency index while the translation variable replaces the time index of the window function of the STFT. The classical wavelet domain noise reduction algorithms perform non-linear thresholding on the wavelet coefficients of the noisy signal. The noise-free signal is obtained through an inverse DWT of the thresholded coefficients. It has been shown that the classical wavelet domain denoising methods achieve near asymptotic minimax performance for functions of certain smoothness classes, when noise is Gaussian [1], [3]. However, these methods breakdown when noise contamination is much more impulsive compared to Gaussian noise and thus are not as effective for denoising FRFs generated from Gaussian measurement errors. A more suitable method for wavelet domain denoising of FRFs contaminated with combined impulsive and Gaussian noise is the *robust wavelet denoising* (RWD) method. This method uses a combination of classical wavelet thresholding methods and median filters for eliminating Gaussian and impulsive noise, respectively [2], [5].

In this paper, an alternative wavelet domain denoising method, similar to the robust wavelet denoising method, is implemented for eliminating FRF noise. The method is based on the *median interpolating pyramid transform* (MIPT). The MIPT fits a parabola to the median of adjacent 3-coefficient bins of the wavelet coefficients at a given course level. This parabola is then used to predict the medians of the next finer scale wavelet coefficients. The true medians of the finer scale wavelet coefficients are calculated and the prediction error becomes the output coefficients of the transform. The decomposition process is repeated in a pyramid structure. These coefficients are thresholded and the decomposition process is reversed resulting in noise free estimates of the FRF [4].

SIMULATIONS

The performance of classical FFT based and wavelet based FRF estimators are evaluated through a simulated analytic 2 degree-of-freedom system defined by the matrices:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, K = \begin{bmatrix} 428400 & -132900 \\ -132900 & 532800 \end{bmatrix} \tag{4}$$

The $H_{21}(\omega)$ system is excited with normal random noise $u(t) \sim \mathcal{N}(0,1)$ and the Gaussian measurement errors at the input and output are assumed to result in 20 and 35 dB signal to noise ratios (SNR), respectively. The true FRF magnitude, the single pass periodogram estimate (SNR=12.7 dB), and the MIPT based estimate (SNR = 18.1 dB) obtained from denoising the single pass periodogram estimate are given in Figure 1. Similar results are obtained for the phase estimates of the FRF.

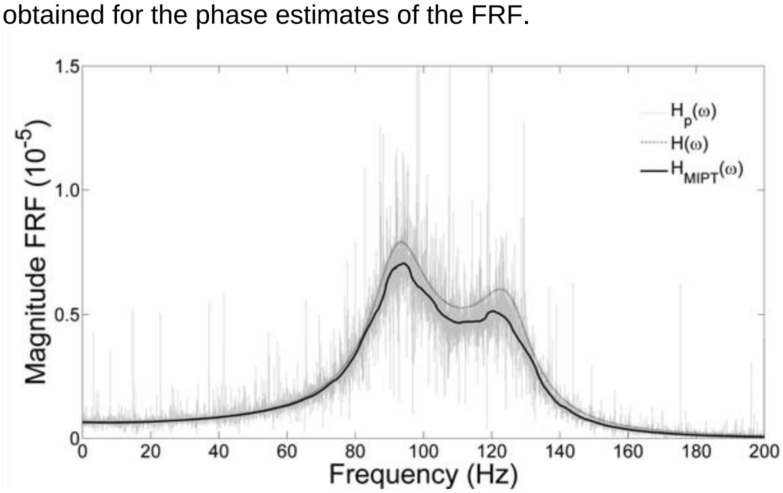

Figure1. The true FRF, the periodogram $H_p(\omega)$, and the MIPT estimates.

CONCLUSIONS

Gaussian measurement noise gets mapped as impulsive noise in the FRFs and can be modeled using alpha-stable distributions. A wavelet domain method that employs median interpolation is implemented for denoising these FRFs with promising results. The MIPT FRF estimates appear to be biased; particularly, the peaks appear to be excessively damped. Nevertheless, MIPT based FRF denoising is shown to have the potential of replacing time domain averaging of periodograms.

REFERENCES

[1] Bodin, P., and Wahlberg, B., "A Frequency Response Estimation Method Based on Smoothing and Thresholding," International Journal of Adaptive Control and Signal Processing, Vol. 12, pp. 407-416, 1998.

[2] Bruce, A. G., Donoho, D. L., Gao, H.-Y., and Martin, R. D., "Denoising and Robust Nonlinear Wavelet Analysis," Proceedings of the SPIE Conference, Wavelet Applications, Vol. 2242, pp. 325-336, 1994.

[3] Donoho, D. L., and Johnstone, I. M., "Ideal Spatial Adaptation by Wavelet Shrinkage," Biometrika, Vol. 81, No. 3, pp. 425-455, 1994.

[4] Donoho, D. L., and Yu, T. P.-Y., "Nonlinear Pyramid Transforms Based on Median Interpolation," SIAM Journal of Mathematical Analysis, Vol. 31, No. 5, pp. 1030-1061, 2000.

[5] Kim, Y. Y., and Hong, J.-C., "Frequency Response Function Estimation via a Robust Wavelet De-noising Method," Journal of Sound and Vibration, Vol. 244, No. 4, pp. 635-649, 2001.