

Parameter Estimation and Control of Nonholonomic Mobile Robots: A Model-Based Approach

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Abstract—This paper presents a mathematical modeling and simulation of a nonholonomic mobile robot for the low-level control regarding the parameter change using model-based design. The model-based design methodology has been a promising technique in automotive and home appliance industries to address and validate the algorithm development, code generation and its deployment on platform. To investigate the electrical and mechanical behavior of the mobile robot, nonlinear mathematical model is constructed and is added to the control loop with the estimator to identify the parameter change. The recursive least square algorithm is used to identify the inertia and the payload of the robot to adapt the dynamic behavior before applying control law. The performance of the model-based approach is investigated on the simulation environment considering the different payloads with DC motor's constraints to track the reference trajectory using PI controller.

Keywords—mobile robot, nonholonomic, low-level control, model-based control, dynamic parameter estimation

I. INTRODUCTION

Autonomous mobile robots have been deployed for a variety of purposes in the manufacturing and industrial setting such as improving material handling efficiency [1] and inspection under hazardous conditions [2]. Precise and robust motion control under varying payloads is crucial for delivery-type mobile robots (alternatively referred to as autonomous transport vehicles, ATV) that generally operate cooperatively as fleets [1]. To perform tasks such as the trajectory tracking, the low-level controller of the ATV should regulate its motion according to the commands from the high-level control module. Thus, reducing tracking errors in the presence of model uncertainty and is major research topic for autonomous mobile robot motion control [3], [4].

Dynamics-based control of mobile robots is difficult in the presence of varying and/or uncertain friction, inertia, and mass during the execution of a task. Therefore, a significant amount of research effort has been devoted to the kinematics based tracking control of mobile robots [5]. In this paper, a recursive least squares method with a forgetting factor [6] is proposed for the online estimation of the pertinent dynamic parameters of a nonholonomic differential autonomous mobile robot. The dynamic model of the robot is updated based

on these estimated parameters in real-time, which is in turn used by the PI type low-level controller for precise trajectory tracking [7].

II. MATHEMATICAL MODELING

In this section, mathematical modeling of nonholonomic mobile robot is considered in terms of actuation, kinematics and dynamics. Fig. 1 shows the actual robot considered for this research (the Evarobot) as well as the generalized coordinates of the system including the notation of the parameters and variables. (see Section II.B)

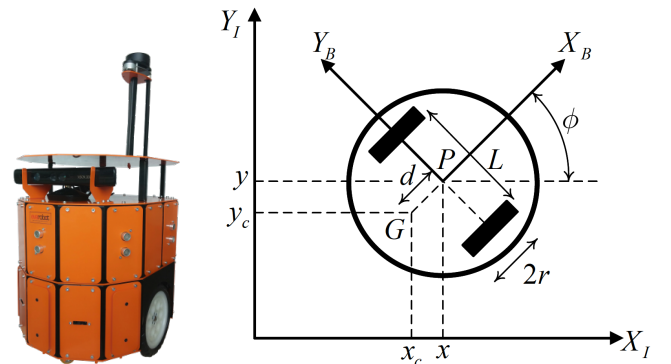


Fig. 1. The Evarobot differential drive robot with a schematic depicting the coordinate systems and the geometric parameters for the differential drive mobile robot.

A. DC motor dynamics

The differential drive mobile robot has two standard (non-steerable) wheels driven by two independent DC motors and two unactuated castor wheels for balance and stability. The independently actuated DC motors provide both translational motion and steering. These motors are gear-coupled to the standard wheels. The governing equation for the DC motor electric circuit are given as:

$$E_a = R_a i_a + L_a \frac{di_a}{dt} + E_b \quad (1)$$

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where E_a , i_a , R_a and L_a are the voltage, current, resistance, and inductance of the armature circuit, respectively, and $E_b = K_b \omega_m$ is the back electromotive force (emf). The governing equation for the mechanical part of the DC motor is:

$$J_m \dot{\omega}_m + b_m \omega_m = \tau_m. \quad (2)$$

where J_m , b_m , ω_m , τ_m are defined as the moment of inertia, the viscous coefficient, the angular velocity and the electrical torque of the motor, respectively.

The electro-mechanical coupling between (1) and (2) is achieved through the motor torque-armature current relationship given as $\tau_m = K_t i_a$ and K_t is the torque constant of the motor. Finally, the torques transmitted from the motors to the wheels are governed by the equations:

$$\tau_i = \eta \tau_m, \quad i = l, r \quad (3)$$

where τ_i is the torque transmitted to the left (l) or right (r) wheels, and η is the gear ratio between the motor and the wheel. Likewise, the transmitted motion from the motor to the wheels is governed by $\omega_m = \eta \omega$ and $\theta_m = \eta \theta$ for the angular velocity and angular displacements, respectively. Table I in Sec. V shows the electrical and mechanical parameters of the DC motor used in the mobile robots considered for this paper.

B. Kinematic and dynamic model of the mobile robot

The posture vector (ξ) and generalized coordinates (\mathbf{q}) of a terrestrial mobile robot is in the form of:

$$\xi = [x \quad y \quad \phi]^T \quad (4a)$$

$$\mathbf{q} = [x_c \quad y_c \quad \phi]^T \quad (4b)$$

where (x, y) and (x_c, y_c) are the x - and y -coordinates of the geometric center (point P) and the mass center (point G) of the robot in the inertial coordinate system, respectively, and ϕ is the heading angle of the robot. The relation between the geometric and the mass center of the mobile robot and its derivative are defined as:

$$x = x_c + d \cos \phi', \quad y = y_c + d \sin \phi' \quad (5a)$$

$$\dot{x} = \dot{x}_c - \dot{\phi}' d \sin \phi', \quad \dot{y} = \dot{y}_c + \dot{\phi}' d \cos \phi', \quad (5b)$$

respectively. The parameter d is defined as the distance between the center of geometry and the center of mass of the robot (see Fig. 1). In case the mass center is not perfectly aligned with the X_B -axis, the heading angle of robot from the center of mass is defined as $\phi' = \phi + \psi$ and $\dot{\phi}' = \dot{\phi}$ where $\psi = \text{atan2}(y - y_c, x - x_c)$ is the fixed angle from the center of geometry to the center of the mass. However, for the differential drive robot utilized in this research it is assumed that the center of mass lies on the robot X_B -axis due to the very small difference in heading angles between the center of mass and the center of geometry of the robot, i.e., $\psi \approx 0.27^\circ$ and therefore it is assumed that $\phi' \approx \phi$.

The angular position and speed of the wheels are given by $\varphi = [\varphi_r \quad \varphi_l]^T$ and $\omega = [\omega_r \quad \omega_l]^T$, respectively. The velocity of the robot in the inertial frame is related to the velocity in the body frame $\mathbf{v} = [v \quad \omega]^T$ through:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \cos \phi & 0 \\ \sin \phi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (6)$$

with $v = (v_r + v_l)/2$ and $\omega = (v_r - v_l)/L$ the linear and angular velocities of the robot, L is the distance between the wheels, $v_l = r \omega_l$ and $v_r = r \omega_r$ are tangential velocities of the left and the right wheels, respectively where r is the wheel radius. Then, one can obtain the wheel angular velocities from the linear and angular velocity of the robot from

$$\begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} = \underbrace{\begin{bmatrix} 1/r & L/(2r) \\ 1/r & -L/(2r) \end{bmatrix}}_{\mathbf{R}_r^w} \begin{bmatrix} v \\ \omega \end{bmatrix} \quad (7)$$

where \mathbf{R}_r^w represents the transformation matrix from robot's linear and angular velocities to the wheel angular velocities.

Combining (5b) and (6) yields the relationship between the center-of-mass and body-frame velocities of the robot as:

$$\dot{\mathbf{q}} = \mathbf{S}(\mathbf{q})\mathbf{v} \quad (8)$$

where the kinematic matrix is defined as

$$\mathbf{S} = \begin{bmatrix} \cos \phi & d \sin \phi \\ \sin \phi & -d \cos \phi \\ 0 & 1 \end{bmatrix} \quad (9)$$

Finally, due to the no lateral motion (side slip) condition, the kinematic constraint equation of the mobile robot becomes

$$-\dot{x}_c \sin \phi + \dot{y}_c \cos \phi + \dot{\phi} d = 0 \quad (10)$$

In the derivation of the dynamic equation for the mobile robot, the method outlined in [8] is utilized. Accordingly, for a n degree-of-freedom (DOF) robot subject to p inputs and m constraints, the general dynamic motion equation is given as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{V}\dot{\mathbf{q}} + \mathbf{f}_d + \mathbf{g} = \mathbf{E}\mathbf{u} - \mathbf{A}^T \boldsymbol{\lambda} \quad (11)$$

where $\mathbf{M} \in \mathbb{R}^{n \times n}$ is the symmetric positive-definite inertia matrix, $\mathbf{V} \in \mathbb{R}^{n \times n}$ is the centripetal and Coriolis force matrix, $\mathbf{f}_d \in \mathbb{R}^n$ represents the friction and damping forces, $\mathbf{g} \in \mathbb{R}^n$ is the gravitational force vector, $\mathbf{E} \in \mathbb{R}^{n \times p}$ is the input transformation matrix, $\mathbf{A} \in \mathbb{R}^{m \times n}$ is the kinematic constraint matrix associated with the nonholonomic constraint equation $\mathbf{A}\dot{\mathbf{q}} = 0$, $\mathbf{q} \in \mathbb{R}^n$ is generalized coordinates, $\mathbf{u} \in \mathbb{R}^p$ is the input vector, and $\boldsymbol{\lambda} \in \mathbb{R}^m$ is the vector of Lagrange multiplier associated with the kinematic constraints.

Ignoring viscous and friction forces, the Lagrangian formulation for the constrained equation of motion of the non-holonomic mobile robot can be expressed as:

$$\frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{q}_k} \right) - \frac{\partial \mathcal{L}}{\partial q_k} = f_k - \sum_{j=1}^m \lambda_j a_{jk} \quad (12)$$

where \mathcal{L} is the Lagrangian, f_k are the generalized forces, λ_j is the j^{th} constraint equation's Lagrange multiplier, and a_{jk} is the k^{th} coefficient of the the j^{th} constraint equation. For the constraint equation given in (10), $j = 1$ and $k = 1, 2, 3$. The Lagrangian for the mobile robot does not include any potential energy terms and can be expressed as $\mathcal{L} = m(\dot{x}_c^2 + \dot{y}_c^2)/2 + (J_r \dot{\phi}^2)/2$ where $m = m_r + m_{pl}$ is the total mass of the mobile robot (m_r) and the payload (m_{pl}), and J_r is the mass moment of inertia of the robot about the z -axis passing through the mass center of the robot. The resulting equations of motion become

$$m\ddot{x}_c - \lambda \sin \phi = \frac{1}{r} (\tau_r + \tau_l) \cos \phi \quad (13a)$$

$$m\ddot{y}_c + \lambda \cos \phi = \frac{1}{r} (\tau_r + \tau_l) \sin \phi \quad (13b)$$

$$J_r \ddot{\phi} + \lambda d = \frac{L}{2r} (\tau_r - \tau_l) \quad (13c)$$

where λ is the Lagrange multiplier, τ_r and τ_l are the driving (input) torques acting on the right and left wheels, respectively. In the derivation of (13), the effects of the rotational dynamics of the wheels are neglected. Eq. (13) can be written in matrix form as:

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{E}\mathbf{u} - \lambda\mathbf{a}^T \quad (14)$$

with the matrices:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_r \end{bmatrix}, \mathbf{E} = \frac{1}{r} \begin{bmatrix} \cos \phi & \cos \phi \\ \sin \phi & \sin \phi \\ \frac{L}{2} & -\frac{L}{2} \end{bmatrix}$$

$$\mathbf{a} = [-\sin \phi \quad \cos \phi \quad d], \mathbf{u} = [\tau_r \quad \tau_l]^T.$$

It is desirable to express the equations of motion in terms of the body coordinates, which can be accomplished by substituting (8) into (14). Pre-multiplying the resulting dynamic equation with \mathbf{S}^T eliminates the Lagrange multipliers since $\mathbf{S}^T \mathbf{a}^T = 0$. The resulting dynamic equation of the mobile robot becomes

$$\tilde{\mathbf{M}}\dot{\mathbf{v}} + \tilde{\mathbf{V}}\mathbf{v} = \tilde{\mathbf{E}}\mathbf{u} \quad (15)$$

where $\tilde{\mathbf{M}} = \mathbf{S}^T \mathbf{M} \mathbf{S}$, $\tilde{\mathbf{V}} = \mathbf{S}^T \mathbf{M} \dot{\mathbf{S}}$, and $\tilde{\mathbf{E}} = \mathbf{S}^T \mathbf{E}$.

For low-level controller design purposes, it is desirable to cast the kinematic and dynamic equations in state-space form. Defining the state vector as $\mathbf{x} = [\mathbf{q}^T \quad \mathbf{v}^T]^T$, the nonlinear state-space model can be expressed as:

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\phi} \\ \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} v \cos \phi + \omega d \sin \phi \\ v \sin \phi - \omega d \cos \phi \\ \omega \\ -\omega^2 d \\ \frac{mdv\omega}{J_t} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{2rJ_t} & \frac{1}{mL} \\ \frac{mL}{2rJ_t} & -\frac{1}{2rJ_t} \end{bmatrix} \begin{bmatrix} \tau_r \\ \tau_l \end{bmatrix} \quad (16)$$

and the total mass moment of inertia is defined as $J_t = J_r + md^2$.

III. PARAMETER ESTIMATION

To accurately model the dynamic behavior of the mobile robot under different payloads, a recursive least square (RLS) algorithm with a forgetting factor (γ) is employed to estimate pertinent mechanical parameters of the system such as the inertia of the robot and it's payload.

A system with unknown parameters can be modeled as

$$\mathbf{y}(k) = \Phi^T(k) \hat{\Theta}(k-1) + \mathbf{e}(k) \quad (17)$$

where $\mathbf{y}(k)$ is the output vector, $\Phi(k)$ is the measurement vector called the regressor, $\hat{\Theta}(k)$ is the unknown parameter vector to be estimated and $\mathbf{e}(k)$ is the prediction error. By minimizing the cost function

$$V(\hat{\Theta}, k) = \frac{1}{2} \sum_{i=1}^k \gamma^{k-i} \left\| \mathbf{y}(i) - \Phi^T(i) \hat{\Theta}(k) \right\|^2 \quad (18)$$

with respect to $\hat{\Theta}$ at time (k), the closed-form solution is obtained as

$$\hat{\Theta}(k) = \left[\sum_{i=1}^k \gamma^{k-i} \Phi(i-1) \Phi^T(i-1) \right]^{-1} \sum_{i=1}^k \gamma^{k-i} \Phi(i-1) \mathbf{y}(i) \quad (19)$$

Since (19) contains matrix inversion, it's computational cost makes it unsuitable for practical, real-time control applications. Alternatively, a recursive scheme based on the matrix inversion lemma [9] can be employed. Accordingly, the unknown parameters are recursively estimated using

$$\hat{\Theta}(k) = \hat{\Theta}(k-1) + \mathbf{K}(k) \mathbf{e}(k) \quad (20)$$

where the first term in (20) is the estimate at the previous time ($k-1$), and the second term is the applied correction. Thus, (20) updates the estimates at each discrete time by correcting the error between the system output and the predicted output. $\mathbf{K}(k)$ in (20) is the gain matrix defined as

$$\mathbf{K}(k) = \frac{\mathbf{P}(k-1) \Phi(k)}{\gamma + \Phi^T(k) \mathbf{P}(k-1) \Phi(k)} \quad (21)$$

with

$$\mathbf{P}(k) = \frac{1}{\gamma} [\mathbf{I} - \mathbf{K}(k)\Phi^T(k)] \mathbf{P}(k-1) \quad (22)$$

being the covariance matrix. Finally, the estimation error is computed as:

$$\mathbf{e}(k) = \mathbf{y}(k) - \Phi^T(k)\hat{\Theta}(k-1). \quad (23)$$

The RLS algorithm updates the parameters at the given discrete time with the selected forgetting factor in the range of $0 < \gamma \leq 1$. The forgetting factor can be chosen large to give less weight to the past errors in (23).

Using the forward-Euler discretization, the dynamics of the mobile robot in (16) can be discretized as:

$$v(k+1) = v(k) - \omega^2(k)t_s d + \frac{t_s}{mr} [\tau_r(k) + \tau_l(k)] \quad (24a)$$

$$\omega(k+1) = \omega(k) + \frac{mt_s d}{J_t} v(k)\omega(k) + \frac{Lt_s}{2rJ_t} [\tau_r(k) - \tau_l(k)] \quad (24b)$$

From (17), the output estimation vector is defined as:

$$\hat{\mathbf{y}}(k) = \Phi^T(k)\hat{\Theta}(k-1). \quad (25)$$

The vectors of the output estimation, the regressor and the parameter estimation can be determined by substituting (24) into (25).

$$\begin{aligned} \underbrace{[v(k+1) \ \omega(k+1)]}_{\hat{\mathbf{y}}} &= \underbrace{[v(k) \ -\omega^2(k) \ \tau_r(k) + \tau_l(k) \ \omega(k) \ v(k)\omega(k) \ \tau_r(k) - \tau_l(k)]}_{\Phi^T} \\ &\quad \times \underbrace{\begin{bmatrix} 1 & t_s \hat{d} & \frac{t_s}{\hat{m}r} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{\hat{d} \hat{m} t_s}{\hat{J}_t} & \frac{L t_s}{2r \hat{J}_t} \end{bmatrix}^T}_{\hat{\Theta}} \end{aligned} \quad (26)$$

Therefore, the mechanical parameters of the mobile robot can be obtained as follows:

$$\hat{d} = \hat{\Theta}_{21}/t_s, \hat{m} = t_s/(r\hat{\Theta}_{31}), \hat{m}_{pl} = \hat{m} - m_r,$$

$$\hat{J}_t = (\hat{m} \hat{d} t_s)/\hat{\Theta}_{52}, \text{ if } \hat{d} \neq 0, \text{ or}$$

$$\hat{J}_t = (L t_s)/(2r \hat{\Theta}_{62}) \text{ independent of the parameter } \hat{d}.$$

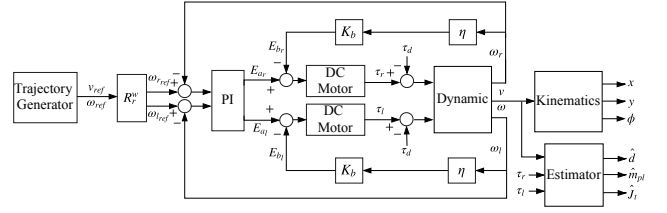


Fig. 2. The block diagram of the feedback loop of the differential drive mobile robot.

IV. CONTROLLER DESIGN

Several model-based design strategies exist for the design of low-level controllers of nonholonomic mobile robots. These strategies are the model-in-the-loop (MIL), the software-in-the-loop (SIL), and the hardware-in-the-loop (HIL). In the low-level controller design presented in this paper, the MIL method is employed for which a mathematical dynamic model of the mobile robot is implemented in the robot's feedback control system. Fig. 2 shows the overall block diagram of the trajectory tracking controller implemented for the nonholonomic mobile robot. The Evarobot (see Fig. 1) is actuated using the DC motors with the mechanical and electrical parameters given in Table I. To follow the reference trajectory, the trajectory generation block provides the reference linear (v_{ref}) and angular velocities (ω_{ref}) of the robot for the predefined path. These robot velocities are transformed to the reference angular velocities for the wheels ($\omega_{i_{ref}}$, $i = l, r$) using the transformation matrix \mathbf{R}_r^w . A PI controller is employed to regulate wheel angular speeds. This PI controller generates the required driving voltages to actuate the DC motors based on the following control equation:

$$E_{a_i} = K_{P_i}(\omega_{i_{ref}} - \omega_i) + K_{I_i} \int (\omega_{i_{ref}} - \omega_i) dt, \quad i = l, r. \quad (27)$$

The PI gains (K_P , K_I) are chosen based on the performance requirements defined as a maximum overshoot less than 25% and a settling time less than 1.5 s. The low-level controller is designed using the SIMULINK software with the following PI gains:

$$K_{P_r} = 14.70, \quad K_{I_r} = 70.25 \quad (28a)$$

$$K_{P_l} = 14.70, \quad K_{I_l} = 70.25. \quad (28b)$$

V. SIMULATION

For the parameter identification process, the forgetting factor (γ) is chosen as 0.999 to reduce the influence of the past estimates on the covariance matrix which is initialized as $\mathbf{P}(0) = \rho_0 \cdot \mathbf{I}_{6 \times 6}$ ($\rho_0 > 0$). The simulation results using the RLS algorithm are given in Fig. 3 and Fig. 5. Fig. 3 shows the errors between the reference and the predicted values in terms of the linear and angular velocities of the robot. The predicted velocities have different convergence times due to the different parameter variation rates as shown in Fig. 5. Identifying the payload (Fig. 4) on the robot is quite fast compared to the

total inertia estimation (Fig. 5) of the robot due to the different parameter variation rate. Fig. 5 also shows the estimate of the total inertia without the distance effect from the payload at the center of mass of the robot [recall that $\hat{J}_t = (Lt_s)/(2r\hat{\Theta}_{62})$]. In general, the identified parameters of the robot converge to the predefined values.

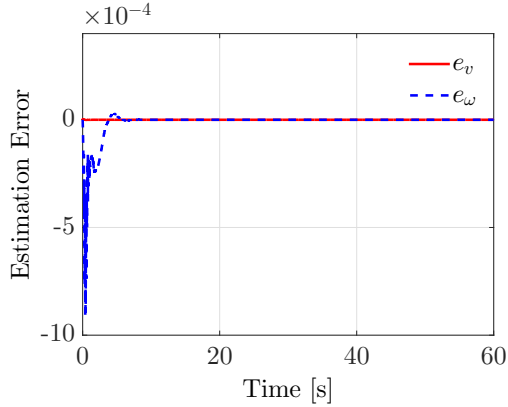


Fig. 3. Linear (e_v) and angular velocity (e_ω) estimation errors.

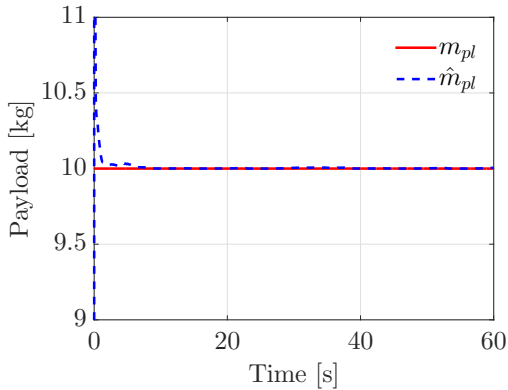


Fig. 4. Time domain plot of payload estimates.

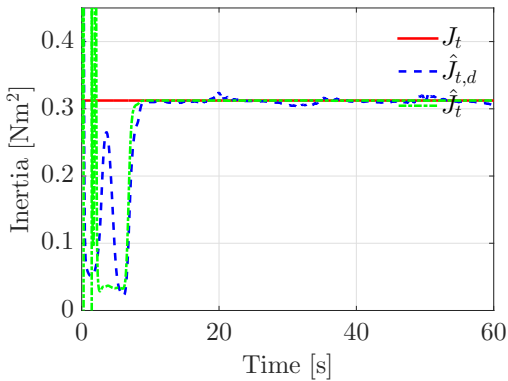


Fig. 5. Inertia estimation with ($\hat{J}_{t,d}$) and without (\hat{J}_t) parameter \hat{d} .

The proposed RLS based estimator (Sec. III) and the low-level controller (Sec. IV) are validated through MATLAB/SIMULINK based numerical simulations. A sampling time (t_s) of 1 ms is used and unmodeled dynamics are

simulated in the form of disturbance torques acting at the output of the DC motors. For this purpose, a band-limited white noise is added into the disturbance torque for 0 kg and 10 kg of the payload under the same initial conditions. The initial conditions are given as $x_0 = 1$ m, $y_0 = 1$ m, $\phi_0 = 63.43^\circ$, $v_0 = 1.4$ m/s, and $\omega_0 = 0$ rad/s. The values of electrical and mechanical parameters of the mobile robot are given in Table I and Table II.

TABLE I. THE MOTOR CHARACTERISTICS.

Quantity	Parameter	Value
torque constant	K_t	18.349 [mNm/A]
back emf constant	K_b	1.921 [mV/rpm]
rotor inertia	J_m	121.8271 [gcm ²]
viscous coefficient	b_m	0.0001 [Nm/(rad/s)]
coil resistance	R_a	0.621532 [Ω]
coil inductance	L_a	0.72 [mH]

TABLE II. THE MECHANICAL PARAMETERS OF THE MOBILE ROBOT.

Parameter	Values
L	0.3175 [m]
r	0.086 [m]
m_r	13.402 [kg]
m_{pl}	0 – 10 [kg]
J_r	0.29172 [kgm ²]
d	$\sqrt{(x - x_c)^2 + (y - y_c)^2}$

Fig. 6 and Fig. 7 show that the robot tends to follow the eight-shaped trajectory [7] with good tracking result in 60 seconds. However, it requires more voltage to generate the torque inputs to drive the wheels for the payload increment. Depending on the DC motor characteristics, the mobile robot can deliver the limited weight to follow the predefined trajectory, which means that above the certain weight, it may take longer time to complete the trajectory tracking or it can be out of the trajectory since the PI controller cannot manage the required voltage and current from the armature circuit due to their saturation. Thus, it is important to estimate the parameter variation in real-time for the autonomous transportation vehicle before it performs decision in high-level control perspective.

VI. CONCLUSION

This paper introduces a model-based approach for the low-level controller design of a nonholonomic, differential drive, autonomous mobile robot. The PI-type controller is augmented with a dynamic parameter estimator for on-line adaptation of the controller to varying payloads. More specifically, a recursive least squares based estimator with a forgetting factor is employed to estimate the payload mass and inertia as well as the center of mass of the combined robot and payload. Several simulations are carried out under two different loading conditions (with and without payload) to demonstrate the effectiveness of the parameter estimator and the controller. In both cases, the parameter estimator is shown to converge to the true values of the dynamic properties of the payload and the controller is shown to precisely track the pre-defined trajectory.

The proposed method is to be implemented on actual Evarobots and prototype ATV's in the future. An improved version of the proposed low-level controller is being developed in the form of an adaptive controller based on the estimated payload parameters.

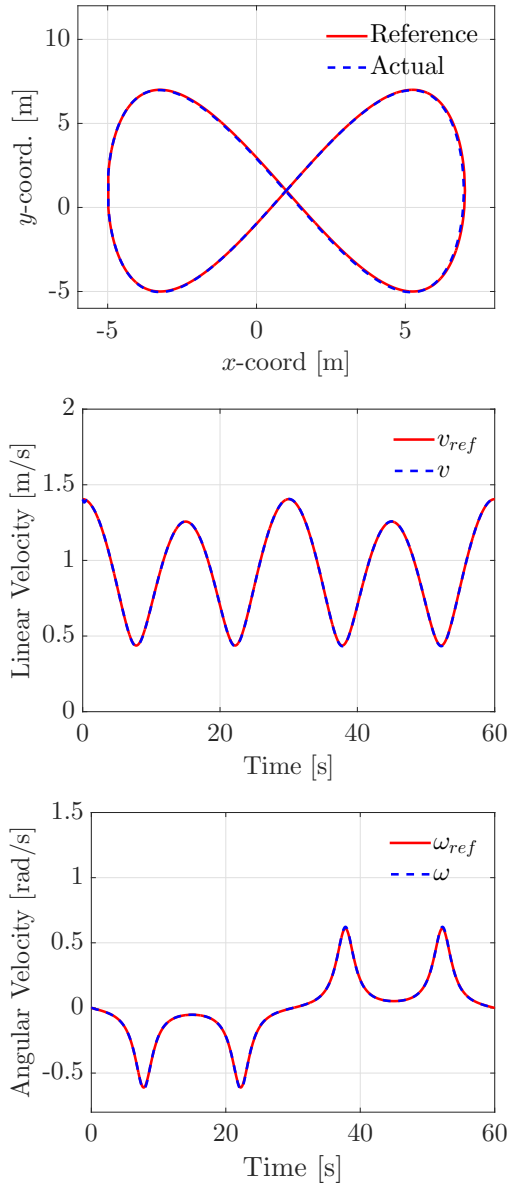


Fig. 6. Simulation results for case 1: No payload ($m_{pl} = 0$ kg).

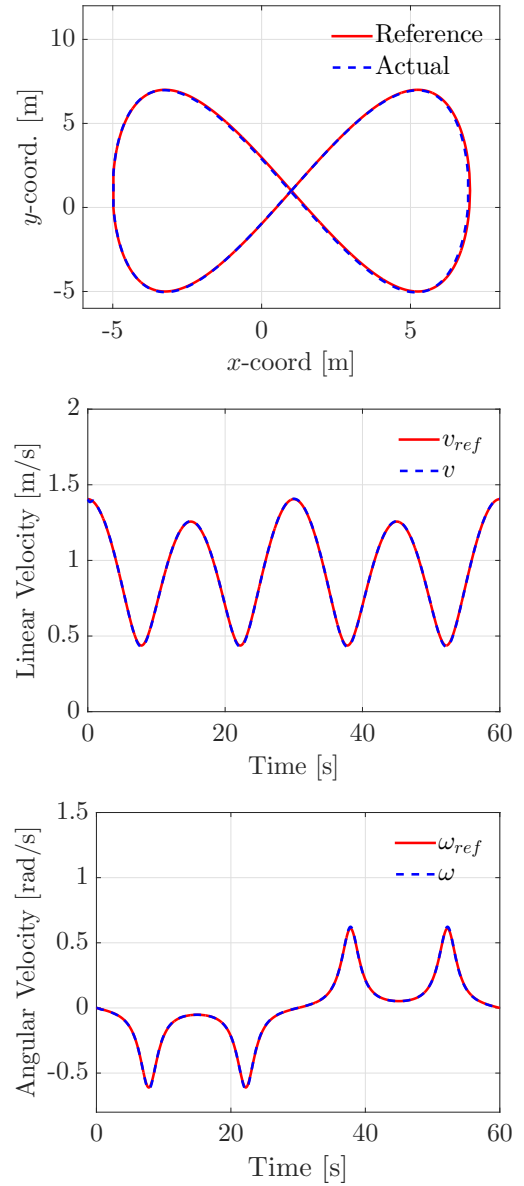


Fig. 7. Simulation results for case 2: With payload ($m_{pl} = 10$ kg)

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